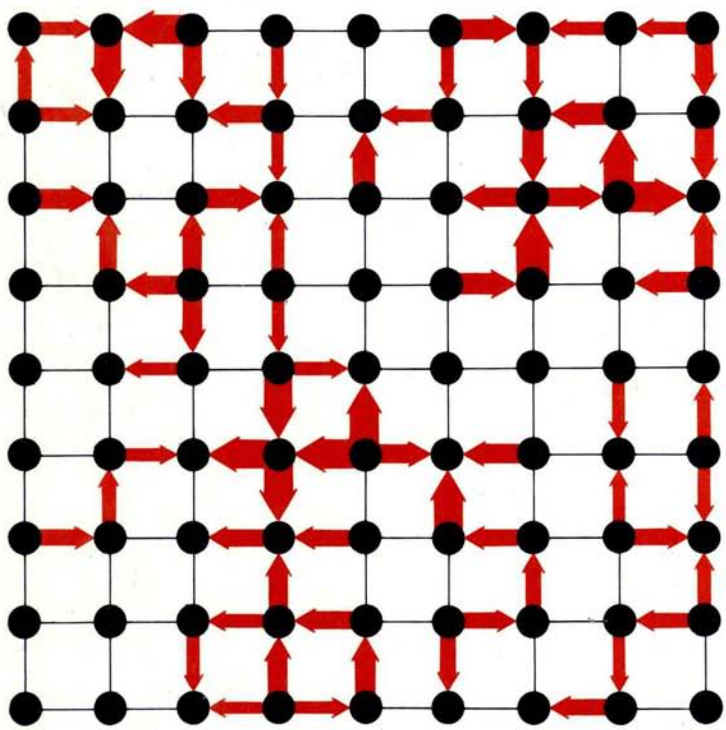
Lattice Gauge Theories and Monte Carlo Simulations

Claudio Rebbi



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PREFACE

Quantized gauge theories play a central role in the description of particle interactions at a fundamental level. As for all quantum fields, quantized gauge systems must be regularized for a proper mathematical definition. Conventional schemes of regularization are based on a perturbative expansion: they can be used to derive a variety of theoretical predictions, but are not suitable for a study of those phenomena which are governed by a strong coupling constant or are of nonperturbative nature. The formulation of gauge theories on a space-time lattice was proposed by Wilson in 1974 to overcome the limitations of perturbative regularization. The lattice regularized quantum gauge theory is a very elegant mathematical construct, which does not rely on any weak coupling for its definition. In particular, strong coupling techniques can be applied and in the strong coupling regime sought for phenomena such as quark confinement are seen to occur. The regularization given by the lattice must, however, eventually be removed by letting the lattice spacing go to zero. In this process the coupling constant gets renormalized and results originally obtained for strong coupling may have to be extrapolated to weak coupling. While on the basis of strong coupling expansions alone this extrapolation may be problematic, information on the outcome of the renormalization procedure can be obtained exploiting another important property of lattice gauge models. These systems, which have many formal analogies with statistical systems used in the description of thermodynamical behavior, like their statistical counterparts can be investigated by numerical methods known as Monte Carlo simulations. Use of the lattice regularization in conjunction with Monte Carlo simulations has produced invaluable results during the last few years, especially for the gauge theory of strong interactions known as Quantum Chromodynamics.

The purpose of this book is to illustrate the lattice formulation of a gauge theory, the techniques which can be used to derive its predictions, with particular emphasis on Monte Carlo simulations, and the major results obtained. A collection of papers, selected so as to provide a global view of what has been accomplished in the field, is presented to the reader. The selection is however not exhaustive of all important contributions: there are many more than could have fit into a volume of reasonable size. A few introductory chapters precede the reprints. Chapter 2, on the formulation of a quantum gauge theory, and Chapter 3, on the Monte Carlo method, have been written to provide the reader who is not familiar with the subject with basic notions about the topics considered in this book. Chapter 4 ought to serve as a guide through the collected papers. Also, additional references can be found in Chapter 4.

I wish to express my gratitude to Dr. Phua and WSPC for inviting me to edit this book, to my colleague Mike Creutz for assistance in the selection of the reprints, to Mrs. Isabell Harrity for her prompt and accurate typing of the manuscript and to the authors and publishers of the journals for their permission to reproduce the papers.

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1. INTRODUCTION

Gauge theories are vital for our understanding of particle systems. Almost all models currently employed to describe particle interactions at a fundamental level make use of the notion of a gauge field. The corresponding quantum systems must be regularized to be given a well defined mathematical meaning and techniques, which are both elegant and powerful, have been developed for the purpose. However, most of these techniques are based on the existence of one or more weak coupling parameters in which the theory can be expanded perturbatively. As such they are not suited for the analysis of phenomena governed by an intrinsically large coupling constant, or, an even worse case, where the behavior at the origin, in the space of complex coupling parameters, is non-analytic. To overcome these difficulties a different method of regularization has been advocated by Wilson [R1]*. It consists of formulating the gauge theory on a discrete lattice of points in Euclidean space-time. The ultraviolet divergences are thus removed and, if the space-time volume of the whole system is made finite to proceed only later to an infinite volume limit, all quantum averages are given by mathematically well-defined expressions, irrespective of the value of the coupling constant. Thus, the lattice regularized theory lends itself to strong coupling expansions, quite analogous to the high temperature expansions for thermodynamical systems, opening a new domain of analytical investigations.

The lattice formulation, however, does not solve all problems of strong coupling. The regularization must eventually be removed

*For notation regarding references see the end of the section.

by letting the spacing between contiguous lattice sites gradually decrease to zero and in the process the coupling parameters must be renormalized: a well-defined continuum limit can be obtained only if these parameters approach a scaling fixed point. The problem remains therefore of demonstrating the existence of such fixed points and of extrapolating there whatever results may have been established within the domain of strong couplings. While expansion techniques may not be powerful enough to achieve these goals, another very useful feature of the lattice formulation succours the theorists. The lattice regularized gauge systems, like their thermodynamical counterparts, allow numerical computations by the technique of importance sampling known as Monte Carlo simulation [A1–4]. The numerical analysis reaches a little further from where the insight obtained by strong coupling expansions seems to fail and bridges the gap between the strong coupling domain and the domain of scaling toward the continuum limit.

Monte Carlo simulations have been extensively applied to the study of lattice gauge theories during the last few years and several important results have been achieved. Some have demonstrated, albeit numerically, the validity of long-standing theoretical conjectures, some have allowed the actual determination of interesting physical observables. The whole lattice approach being motivated by the instance of getting new clues to strong coupling phenomena, it is not surprising that the majority, as well as the most relevant, of Monte Carlo results have been obtained for the gauge theory of strong interactions known as Quantum Chromodynamics (QCD).

The purpose of this book is to gather together a collection of reprints providing a path through the formulation of a gauge theory on a lattice all the way to some of the most recent Monte Carlo results for QCD. Once the basic ideas underlying the lattice regularization and the Monte Carlo numerical technique are assimilated, it is really straightforward to proceed through the literature in this field. However, the fundamental notions are not found all together, other than in review articles or lecture notes. Hence, to facilitate the task of the reader not already familiar with the subject, two introductory chapters, one on the definition of a lattice gauge system, the other on the Monte Carlo method, precede the reprints. A detailed account of the results achieved would be too cumbersome as an introductory chapter, and these are best learnt from the original articles. Thus, only a rather concise guide through the literature concludes, in Chapter 4, the introductory notes.

Finally, a word about references. Those to be found reprinted in this volume are indicated by an R followed by the serial number. Additional references are indicated by A with a serial number and listed at the end of the introductory chapters. Given the very large number of articles published on the topic of lattice gauge theories and, more specifically, on Monte Carlo simulations of such systems, any collection of reprints which can be put together in a volume of manageable size is bound to exclude a large number of valuable works. No judgement of quality is implied in the selection made in this book, which has rather been based on the criterion of providing a continuous and possibly smooth pathway through the subject of lattice gauge theories and Monte Carlo simulations. To a lesser extent these remarks apply also to the list of references appended at the end of the introductory chapters, which in no way claims to be complete, or exhaustive of the relevant contributions to the field.