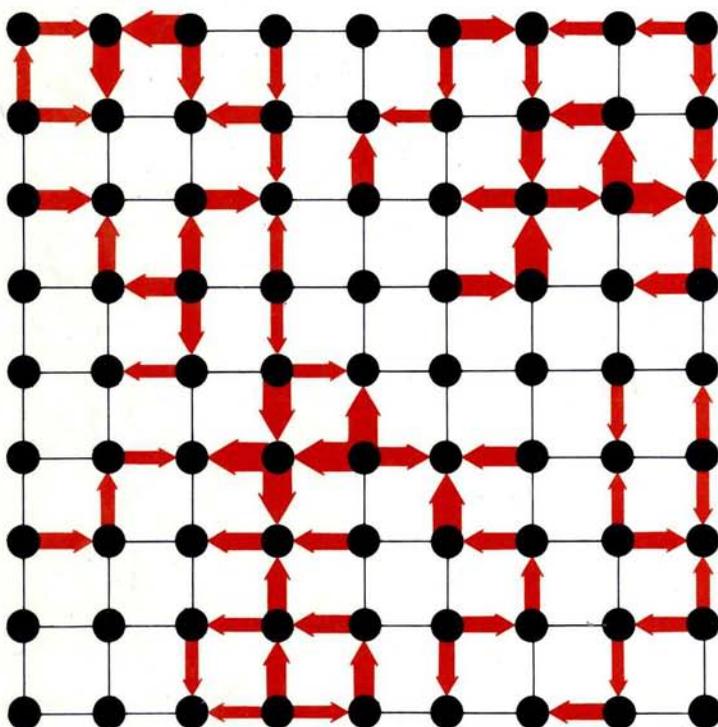

Lattice Gauge Theories and Monte Carlo Simulations

Claudio Rebbi



World Scientific

**LATTICE GAUGE THEORIES
AND
MONTE CARLO SIMULATIONS**

This page is intentionally left blank

Lattice Gauge Theories and Monte Carlo Simulations

Claudio Rebbi

Brookhaven National Laboratory



World Scientific

**World Scientific Publishing Co Pte Ltd
P O Box 128
Farrer Road
Singapore 9128**

The author and publisher are indebted to the original authors and publishers of the various journals for their assistance and permission to reproduce the selected papers found in this volume.

© 1983 by World Scientific Publishing Co Pte Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the publisher.

**ISBN 9971-950-70-7
9971-950-71-5 pbk**

Printed in Singapore by Richard-Clay (S. E. Asia) Pte. Ltd.

PREFACE

Quantized gauge theories play a central role in the description of particle interactions at a fundamental level. As for all quantum fields, quantized gauge systems must be regularized for a proper mathematical definition. Conventional schemes of regularization are based on a perturbative expansion: they can be used to derive a variety of theoretical predictions, but are not suitable for a study of those phenomena which are governed by a strong coupling constant or are of nonperturbative nature. The formulation of gauge theories on a space-time lattice was proposed by Wilson in 1974 to overcome the limitations of perturbative regularization. The lattice regularized quantum gauge theory is a very elegant mathematical construct, which does not rely on any weak coupling for its definition. In particular, strong coupling techniques can be applied and in the strong coupling regime sought for phenomena such as quark confinement are seen to occur. The regularization given by the lattice must, however, eventually be removed by letting the lattice spacing go to zero. In this process the coupling constant gets renormalized and results originally obtained for strong coupling may have to be extrapolated to weak coupling. While on the basis of strong coupling expansions alone this extrapolation may be problematic, information on the outcome of the renormalization procedure can be obtained exploiting another important property of lattice gauge models. These systems, which have many formal analogies with statistical systems used in the description of thermodynamical behavior, like their statistical counterparts can be investigated by numerical methods known as Monte Carlo simulations. Use of the lattice regularization in conjunction with Monte Carlo simulations has produced invaluable results during the last few years, especially for the gauge theory of strong interactions known as Quantum Chromodynamics.

The purpose of this book is to illustrate the lattice formulation of a gauge theory, the techniques which can be used to derive its *predictions, with particular emphasis on Monte Carlo simulations*, and the major results obtained. A collection of papers, selected so as to provide a global view of what has been accomplished in

the field, is presented to the reader. The selection is however not exhaustive of all important contributions: there are many more than could have fit into a volume of reasonable size. A few introductory chapters precede the reprints. Chapter 2, on the formulation of a quantum gauge theory, and Chapter 3, on the Monte Carlo method, have been written to provide the reader who is not familiar with the subject with basic notions about the topics considered in this book. Chapter 4 ought to serve as a guide through the collected papers. Also, additional references can be found in Chapter 4.

I wish to express my gratitude to Dr. Phua and WSPC for inviting me to edit this book, to my colleague Mike Creutz for assistance in the selection of the reprints, to Mrs. Isabell Harrity for her prompt and accurate typing of the manuscript and to the authors and publishers of the journals for their permission to reproduce the papers.

CONTENTS

<i>Preface</i>	v
----------------	---

Introductory Chapters

1	Introduction	1
2	Formulation of a Quantized Lattice Gauge Theory	4
3	The Monte Carlo Method	17
4	A Guide through the Literature	23
	Additional References	38

REPRINTED PAPERS

R1	Confinement of quarks by K. G. Wilson <i>Phys. Rev. D</i> 10 (1974) 2445 – 2459	45
R2	Duality in generalized Ising models and phase transitions without local order parameters by Franz J. Wegner <i>J. Math. Phys.</i> 10 (1971) 2259 – 2272	60
R3	Gauge fields on a lattice. I. General outlook by R. Balian, J. M. Drouffe, and C. Itzykson <i>Phys. Rev. D</i> 10 (1974) 3376 – 3395	74
R4	Gauge fields on a lattice. II. Gauge invariant Ising model by R. Balian, J. M. Drouffe, and C. Itzykson <i>Phys. Rev. D</i> 11 (1975) 2098 – 2103	94
R5	Gauge fields on a lattice. III. Strong-coupling expansions and transition points by R. Balian, J. M. Drouffe, and C. Itzykson <i>Phys. Rev. D</i> 11 (1975) 2104 – 2119 Errata, <i>Phys. Rev. D</i> 19 (1979) 2514 – 2515	100

- R6 Hamiltonian formulation of Wilson's lattice gauge theories
by John Kogut and Leonard Susskind
Phys. Rev. D **11** (1975) 395 – 408 118
- R7 Gauge fixing, the transfer matrix, and confinement on a lattice
by Michael Creutz
Phys. Rev. D **15** (1977) 1128 – 1136 132
- R8 Gauge theory on a random lattice
by N. H. Christ, R. Friedberg, and T. D. Lee
Nucl. Phys. B **210** [FS6] (1982) 310 – 336 141
- R9 Weights of links and plaquettes in a random lattice
by N. H. Christ, R. Friedberg, and T. D. Lee
Nucl. Phys. B **210** [FS6] (1982) 337 – 346 168
- R10 Experiments with a gauge-invariant Ising system
by Michael Creutz, Laurence Jacobs, and Claudio Rebbi
Phys. Rev. Lett. **42** (1979) 1390 – 1393 178
- R11 Monte Carlo study of Abelian lattice gauge theories
by Michael Creutz, Laurence Jacobs, and Claudio Rebbi
Phys. Rev. D **20** (1979) 1915 – 1922 182
- R12 Phase transition in four-dimensional compact QED
by B. Lautrup and M. Nauenberg
Phys. Lett. **95B** (1980) 63 – 66 190
- R13 Topological excitations and Monte Carlo simulation of Abelian gauge theory
by T. A. DeGrand and Doug Toussaint
Phys. Rev. D **22** (1980) 2478 – 2489 194

-
- R14 Confinement and the critical dimensionality of space-time
by Michael Creutz
Phys. Rev. Lett. **43** (1979) 553 – 556 206
- R15 Phase structure of non-Abelian lattice gauge theories
by Claudio Rebbi
Phys. Rev. **D21** (1980) 3350 – 3359 210
- R16 A fast algorithm for Monte Carlo simulations of 4-d lattice gauge theories with finite groups
by Gyan Bhanot, Christian B. Lang, and Claudio Rebbi
Computer Physics Communications **25** (1982) 275 – 287 220
- R17 Ultraviolet behavior of non-Abelian gauge theories
by David J. Gross and Frank Wilczek
Phys. Rev. Lett. **30** (1973) 1343 – 1346 233
- R18 Reliable perturbative results for strong interactions?
by H. David Politzer
Phys. Rev. Lett. **30** (1973) 1346 – 1349 237
- R19 The connection between the Λ parameters of lattice and continuum QCD
by Anna Hasenfratz and Peter Hasenfratz
Phys. Lett. **93B** (1980) 165 – 169 241
- R20 Relationship between lattice and continuum definitions of the gauge-theory coupling
by Roger Dashen and David J. Gross
Phys. Rev. **D23** (1981) 2340 – 2348 246
- R21 Quantum-Chromodynamic β function at intermediate and strong coupling
by John B. Kogut, Robert B. Pearson, and Junko Shigemitsu
Phys. Rev. Lett. **43** (1979) 484 – 486 255

- R22 Monte Carlo study of quantized SU(2) gauge theory
by Michael Creutz
Phys. Rev. D **D21** (1980) 2308 – 2315 258
- R23 Asymptotic-freedom scales
by Michael Creutz
Phys. Rev. Lett. **45** (1980) 313 – 316 266
- R24 String tension in SU(3) lattice gauge theory
by E. Pietarinen
Nucl. Phys. **B190** [FS3] (1981) 349 – 356 270
- R25 Estimate of the relation between scale parameters and the string tension by strong coupling methods
by Gernot Münster and Peter Weisz
Phys. Lett. **96B** (1980) 119 – 122 278
- R26 Errata – G. Münster and P. Weisz, Estimate of the relation between scale parameters and the string tension by strong coupling methods,
Phys. Lett. **96B** (1980) 119.
Phys. Lett. **100B** (1981) 519 282
- R27 A Monte Carlo study of SU(2) Yang-Mills theory at finite temperature
by Larry D. McLerran and Benjamin Svetitsky
Phys. Lett **98B** (1981) 195 – 198 283
- R28 Monte Carlo study of SU(2) gauge theory at finite temperature
by J. Kuti, J. Polónyi, and K. Szlachányi
Phys. Lett. **98B** (1981) 199 – 204 287
- R29 High temperature SU(2) gluon matter on the lattice
by J. Engels, F. Karsch, H. Satz, and I. Montvay
Phys. Lett. **101B** (1981) 89 – 94 293

-
- R30 SU(2) string tension, glueball mass and interquark potential by Monte Carlo computations
by G. Bhanot and C. Rebbi
Nucl. Phys. B **180** [FS2] (1981) 469 – 482 299
- R31 Heavy quark potential in SU(2) lattice gauge theory
by John D. Stack
Phys. Rev. D **27** (1983) 412 – 420 313
- R32 Potential and restoration of rotational symmetry in SU(2) lattice gauge theory
by C. B. Lang and C. Rebbi
Phys. Lett. **115B** (1982) 137 – 142 322
- R33 Monte Carlo estimates of the SU(2) mass gap
by B. Berg, A. Billoire and C. Rebbi
Ann. Phys. **142** (1982) 185 – 215 328
- R34 On the masses of the glueballs in pure SU(2) lattice gauge theory
by M. Falcioni, E. Marinari, M. I. Paciello, G. Parisi, F. Rapuano, B. Taglienti and Zhang Yi-Cheng
Phys. Lett. **110B** (1982) 295 – 298 359
- R35 Numerical estimate of the SU(3) glueball mass
by B. Berg and A. Billoire
Phys. Lett. **113B** (1982) 65 – 68 363
- R36 SU(3) lattice Monte Carlo calculation of the glueball mass spectrum
by K. Ishikawa, M. Teper and G. Schierholz
Phys. Lett. **116B** (1982) 429 – 433 367
- R37 Determination of the correlation length by varying the boundary conditions in lattice gauge theories
by K. H. Mütter and K. Schilling
Phys. Lett. **117B** (1982) 75 – 80 372

- R38 A property of electric and magnetic flux in non-Abelian gauge theories
by G. 't Hooft
Nucl. Phys. B513 (1979) 141 – 160 378
- R39 Monopoles, vortices and confinement
by G. Mack and E. Pietarinen
Nucl. Phys. B205 [FS5] (1982) 141 – 167 399
- R40 Preliminary evidence for $U_A(1)$ breaking in QCD from lattice calculations
by P. DiVecchia, K. Fabricius, G. C. Rossi and G. Veneziano
Nucl. Phys. B192 (1981) 392 – 408 426
- R41 Numerical checks of the lattice definition independence of topological charge fluctuations
by P. DiVecchia, K. Fabricius, G. C. Rossi and G. Veneziano
Phys. Lett. 108B (1982) 323 – 326 443
- R42 The action of $SU(N)$ lattice gauge theory in terms of the heat kernel on the group manifold
by P. Menotti and E. Onofri
Nucl. Phys. B190 [FS3] (1981) 288 – 300 447
- R43 The transition from strong coupling to weak coupling in the $SU(2)$ lattice gauge theory
by C. B. Lang, C. Rebbi, P. Salomonson and B. S. Skagerstam
Phys. Lett. 101B (1981) 173 – 179 460
- R44 The phase structure of $SU(N)/Z(N)$ lattice gauge theories
by I. G. Halliday and A. Schwimmer
Phys. Lett. 101B (1981) 327 – 331 467
- R45 First-order phase transition in four-dimensional $SO(3)$ lattice gauge theory
by J. Greensite and B. Lautrup
Phys. Rev. Lett. 47 (1981) 9 – 11 472

-
- R46 Variant actions and phase structure in lattice gauge theory
by Gyan Bhanot and Michael Creutz
Phys. Rev. D **D24** (1981) 3212 – 3217 475
- R47 Phase diagrams for coupled spin-gauge systems
by Michael Creutz
Phys. Rev. D **D21** (1980) 1006 – 1012 481
- R48 The phase structure of a non-Abelian gauge Higgs field system
by C. B. Lang, C. Rebbi and M. Virasoro
Phys. Lett. **104B** (1981) 294 – 300 488
- R49 Lattice fermions: species doubling, chiral invariance and the triangle anomaly
by Luuk H. Karsten and Jan Smit
Nucl. Phys. **B185** (1981) 20 – 40 495
- R50 Absence of neutrinos on a lattice. (I). Proof by homotopy theory
by H. B. Nielsen and M. Ninomiya
Nucl. Phys. **B185** (1981) 20 – 40
Errata, *Nucl. Phys.* **B195** (1982) 541 – 542 533
- R51 Effective Lagrangian and dynamical symmetry breaking in strongly coupled lattice QCD
by N. Kawamoto and J. Smit
Nucl. Phys. **B192** (1981) 100 – 124 556
- R52 The strong-coupling limit of gauge theories with fermions on a lattice
by Hannah Kluberg-Stern, André Morel and Bengt Petersson
Phys. Lett. **114B** (1982) 152 – 156 581
- R53 A proposal for Monte Carlo simulations of fermionic systems
by F. Fucito, E. Marinari, G. Parisi and C. Rebbi
Nucl. Phys. **B180** [FS2] (1981) 369 – 377 586

- R54 Monte Carlo simulation of the massive Schwinger model
by E. Marinari, G. Parisi and C. Rebbi
Nucl. Phys. B **190** [FS3] (1981) 734 – 750 595
- R55 Method for performing Monte Carlo calculations for systems with fermions
by D. J. Scalapino and R. L. Sugar
Phys. Rev. Lett. **46** (1981) 519 – 521 612
- R56 Stochastic method for the numerical study of lattice fermions
by Julius Kuti
Phys. Rev. Lett. **49** (1982) 183 – 186 615
- R57 Numerical estimates of hadronic masses in a pure SU(3) gauge theory
by H. Hamber and G. Parisi
Phys. Rev. Lett. **47** (1981) 1792 – 1795 619
- R58 Computer estimates for meson masses in SU(2) lattice gauge theory
by E. Marinari, G. Parisi and C. Rebbi
Phys. Rev. Lett. **47** (1981) 1795 – 1799 623
- R59 Spectroscopy in a lattice gauge theory
by H. Hamber, E. Marinari, G. Parisi and C. Rebbi
Phys. Lett. **108B** (1982) 314 – 316 628
- R60 Monte Carlo evaluation of hadron masses in lattice gauge theories with fermions
by Don Weingarten
Phys. Lett. **109B** (1982) 57 – 62 631
- R61 Hopping parameter expansion for the meson spectrum in SU(3) lattice QCD
by A. Hasenfratz, Z. Kunszt, P. Hasenfratz and C. B. Lang
Phys. Lett. **101B** (1982) 289 – 294 637
- R62 Hadron spectroscopy in lattice QCD
by F. Fucito, G. Martinelli, C. Omero, G. Parisi, R. Petronzio and R. Rapuano
Nucl. Phys. B **210** [FS6] (1982) 407 – 421 643

Introductory Chapters

**MONTE CARLO COMPUTATIONS
IN LATTICE GAUGE THEORIES**

This page is intentionally left blank

1. INTRODUCTION

Gauge theories are vital for our understanding of particle systems. Almost all models currently employed to describe particle interactions at a fundamental level make use of the notion of a gauge field. The corresponding quantum systems must be regularized to be given a well defined mathematical meaning and techniques, which are both elegant and powerful, have been developed for the purpose. However, most of these techniques are based on the existence of one or more weak coupling parameters in which the theory can be expanded perturbatively. As such they are not suited for the analysis of phenomena governed by an intrinsically large coupling constant, or, an even worse case, where the behavior at the origin, in the space of complex coupling parameters, is non-analytic. To overcome these difficulties a different method of regularization has been advocated by Wilson [R1]*. It consists of formulating the gauge theory on a discrete lattice of points in Euclidean space-time. The ultraviolet divergences are thus removed and, if the space-time volume of the whole system is made finite to proceed only later to an infinite volume limit, all quantum averages are given by mathematically well-defined expressions, irrespective of the value of the coupling constant. Thus, the lattice regularized theory lends itself to strong coupling expansions, quite analogous to the high temperature expansions for thermodynamical systems, opening a new domain of analytical investigations.

The lattice formulation, however, does not solve all problems of strong coupling. The regularization must eventually be removed

*For notation regarding references see the end of the section.

by letting the spacing between contiguous lattice sites gradually decrease to zero and in the process the coupling parameters must be renormalized: a well-defined continuum limit can be obtained only if these parameters approach a scaling fixed point. The problem remains therefore of demonstrating the existence of such fixed points and of extrapolating there whatever results may have been established within the domain of strong couplings. While expansion techniques may not be powerful enough to achieve these goals, another very useful feature of the lattice formulation succours the theorists. The lattice regularized gauge systems, like their thermodynamical counterparts, allow numerical computations by the technique of importance sampling known as Monte Carlo simulation [A1–4]. The numerical analysis reaches a little further from where the insight obtained by strong coupling expansions seems to fail and bridges the gap between the strong coupling domain and the domain of scaling toward the continuum limit.

Monte Carlo simulations have been extensively applied to the study of lattice gauge theories during the last few years and several important results have been achieved. Some have demonstrated, albeit numerically, the validity of long-standing theoretical conjectures, some have allowed the actual determination of interesting physical observables. The whole lattice approach being motivated by the instance of getting new clues to strong coupling phenomena, it is not surprising that the majority, as well as the most relevant, of Monte Carlo results have been obtained for the gauge theory of strong interactions known as Quantum Chromodynamics (QCD).

The purpose of this book is to gather together a collection of reprints providing a path through the formulation of a gauge theory on a lattice all the way to some of the most recent Monte Carlo results for QCD. Once the basic ideas underlying the lattice regularization and the Monte Carlo numerical technique are assimilated, it is really straightforward to proceed through the literature in this field. However, the fundamental notions are not found all together, other than in review articles or lecture notes. Hence, to facilitate the task of the reader not already familiar with the subject, two introductory chapters, one on the

definition of a lattice gauge system, the other on the Monte Carlo method, precede the reprints. A detailed account of the results achieved would be too cumbersome as an introductory chapter, and these are best learnt from the original articles. Thus, only a rather concise guide through the literature concludes, in Chapter 4, the introductory notes.

Finally, a word about references. Those to be found reprinted in this volume are indicated by an R followed by the serial number. Additional references are indicated by A with a serial number and listed at the end of the introductory chapters. Given the very large number of articles published on the topic of lattice gauge theories and, more specifically, on Monte Carlo simulations of such systems, any collection of reprints which can be put together in a volume of manageable size is bound to exclude a large number of valuable works. No judgement of quality is implied in the selection made in this book, which has rather been based on the criterion of providing a continuous and possibly smooth pathway through the subject of lattice gauge theories and Monte Carlo simulations. To a lesser extent these remarks apply also to the list of references appended at the end of the introductory chapters, which in no way claims to be complete, or exhaustive of the relevant contributions to the field.